Stochastic Phase Dynamics in Turbulent Regimes

→ Some thoughts toward a unified model

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Outline

- Some observations:
  - Proliferating zoology of QH-mode states
  - Its the cross-phase, …
  - Questions
- Re-visiting MHD Turbulent ELM-free states:
  - Findings
  - The phase, again
- Toward a unified scenario

Some Observations

- QH states are proliferating…
  - EHO (Garofalo, et. al.)
  - Wide pedestal, turbulent (may coexist with EHO) (Burrell, Chen)
  - LCO (Barada)
- Strong ExB shear is common element to all
- Cross phase dynamics is <u>critical</u>:
  - Phase evolves, <u>dynamically</u>
  - Contrast fixed value, as familiar in QL models

Some Key Questions:

- If EHO ←→ coherent phase dynamics, slips, locking

  then

  Turbulent QH ←→ stochastic cross phase evolution?

  ⇔ existing work suggests yes.

- How connect/unify coherent, turbulent regimes?

N.B. Easy to see that strong V_E' is beneficial in both scenarios

I) Basic Notions

ELM Bursts vs Turbulence:

Consequence of Stochastic Phase Dynamics

→ See P.W. Xi, X.-Q. Xu, P.D.; PRL 2014
P.W. Xi, X.-Q. Xu, P.D.; PoP 2014, 2015
Z.B. Guo, P.D. PRL 2015

Model and equilibrium in BOUT++

- **3-field model for nonlinear ELM simulations**
  - ✓ **Including essential physics for the onset of ELMs**

Peeling-ballooning instability

Resistivity

Hyper-resistivity

Ion diamagnetic effect

$$\frac{d\varpi}{dt} = \boxed{B\nabla_\parallel J_\parallel + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \widetilde{P}} + \mu_{i,\parallel}\partial_\parallel^2 \varpi$$

$$\frac{d\widetilde{P}}{dt} + \mathbf{V}_E \cdot \nabla P_0 = 0$$

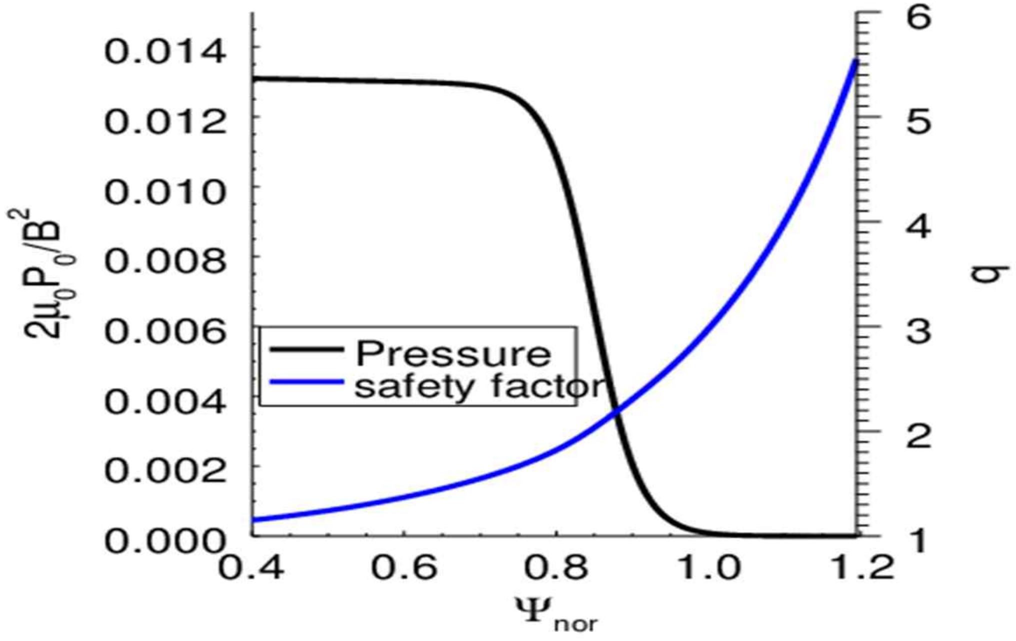
hyper resistivity

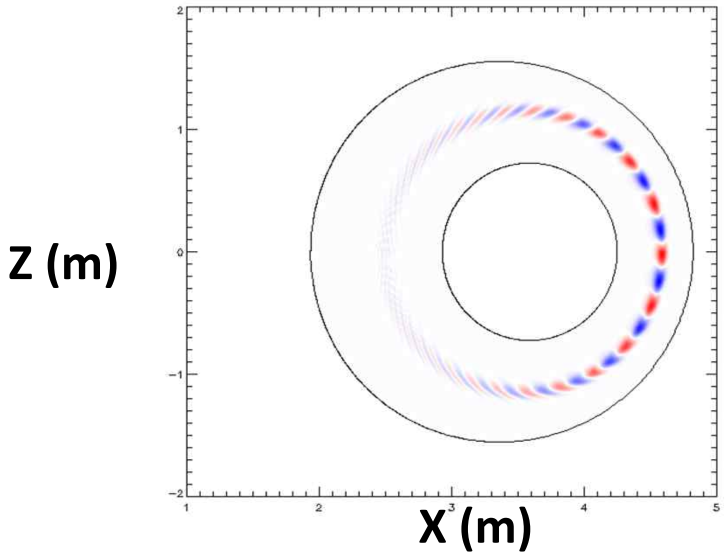
$$\frac{\partial A_\parallel}{\partial t} + \partial_\parallel \phi_T = \boxed{\frac{\eta}{\mu_0}\nabla_\perp^2 A_\parallel} - \boxed{\frac{\eta_H}{\mu_0}\nabla_\perp^4 A_\parallel}$$

$$\varpi = \frac{m_i n_0}{B}\left(\nabla_\perp^2 \phi + \boxed{\frac{1}{en_0}\nabla_\perp^2 \widetilde{P}_i}\right)$$

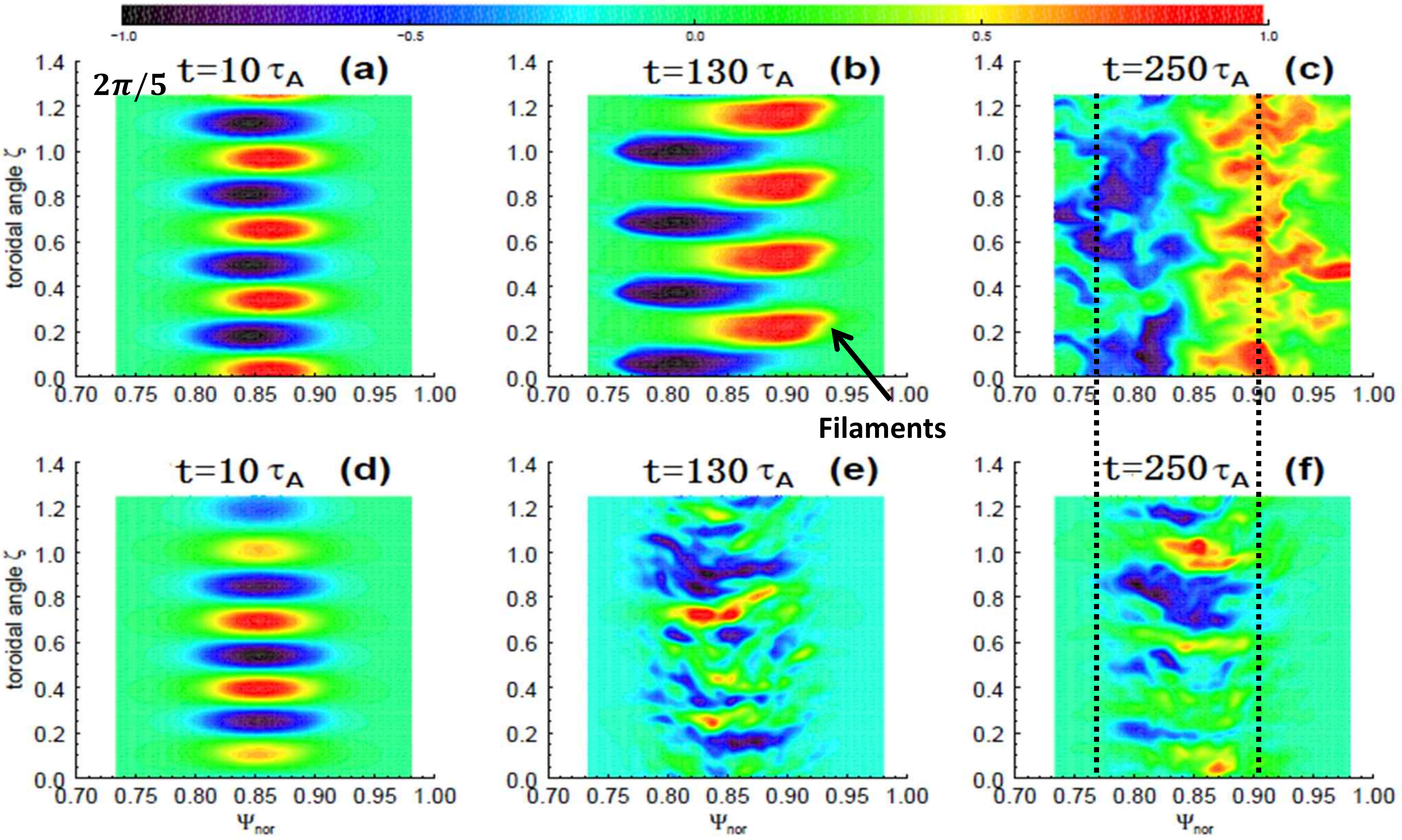
→ ZF feedback

$$d/dt = \partial/\partial t + \mathbf{V}_{ET}\cdot\nabla, \mathbf{V}_{ET} = \frac{1}{B}\mathbf{b}_0 \times \nabla\phi_T, \phi_T = \phi_0 + \phi, \nabla_\parallel f = B\partial_\parallel \frac{f}{B}, \partial_\parallel = \partial_\parallel^0 + \delta\mathbf{b}\cdot\nabla, \delta\mathbf{b} = \frac{1}{B}\nabla A_\parallel \times \mathbf{b}_0, J_\parallel = J_{\parallel 0} + \widetilde{J}_\parallel, \widetilde{J}_\parallel = -\nabla_\perp^2 A_\parallel / \mu_0$$





Contrast perturbation evolution



Filaments

Single Mode

Multiple Mode

Linear phase **Early nonlinear phase** **Late nonlinear phase**

- **Single mode: Filamentary structure is generated by linear instability;**
- **Multiple modes: Linear mode structure is disrupted by nonlinear mode interaction and no filamentary structure appears – <u>turbulent state</u>**

➔ reduced tendency to penetrate outwards

→ Cross Phase Dynamics Regulates
Outcome of P.-B. Evolution

Peeling-Ballooning Perturbation Amplification is set by Coherence of Cross-Phase

i.e. schematic P.B. energy equation:

$$\frac{\partial}{\partial t}E_k = \langle\tilde{\phi}2\hat{b}_0 \times \vec{k} \cdot \nabla\tilde{P}\rangle_{\vec{k}}$$

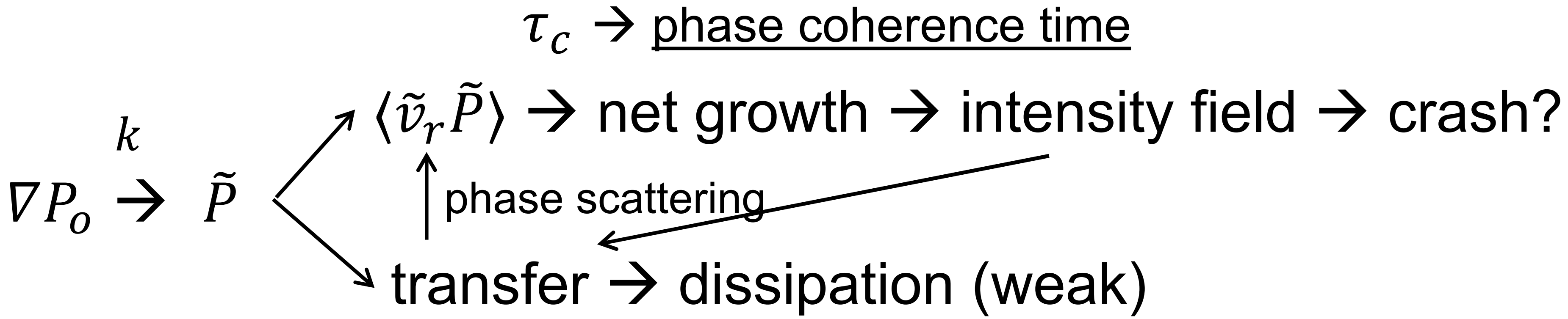
$\sim \langle\tilde{v}_r\tilde{P}\rangle$ → energy release from $\nabla\langle P\rangle$
→ quadratic

$$+\sum_{\vec{k}',\vec{k}''}\tau_{c\vec{k}}C(\vec{k}',\vec{k}'')E_{\vec{k}'}E_{\vec{k}''} - \sum_{\vec{k}'}\tau_{c\vec{k}+\vec{k}'}C(\vec{k}',\vec{k})E_{\vec{k}'}E_{\vec{k}}$$ - dissipation

nonlinear mode-mode coupling → quartic

NL effects
- energy couplings to transfer energy (weak)
- response scattering to de-correlate $\tilde{\phi}$, $\tilde{P}$ ➔ regulate drive

Growth Regulated by Phase Scattering



τ_c → phase coherence time

$\langle \tilde{v}_r \tilde{P} \rangle$ → net growth → intensity field → crash?

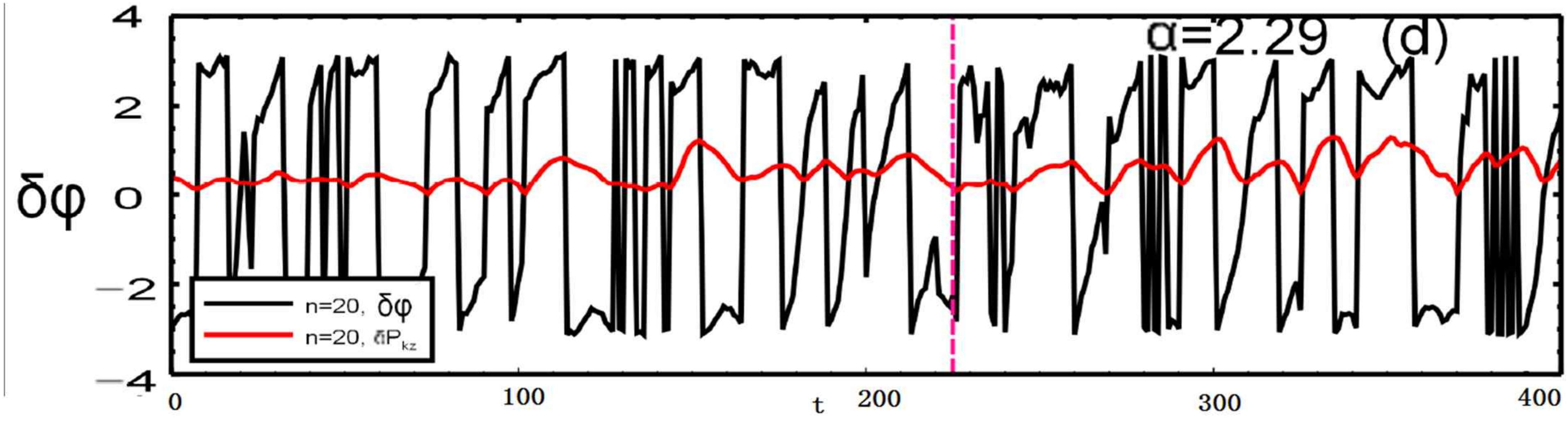
$\nabla P_o \xrightarrow{k} \tilde{P}$

phase scattering

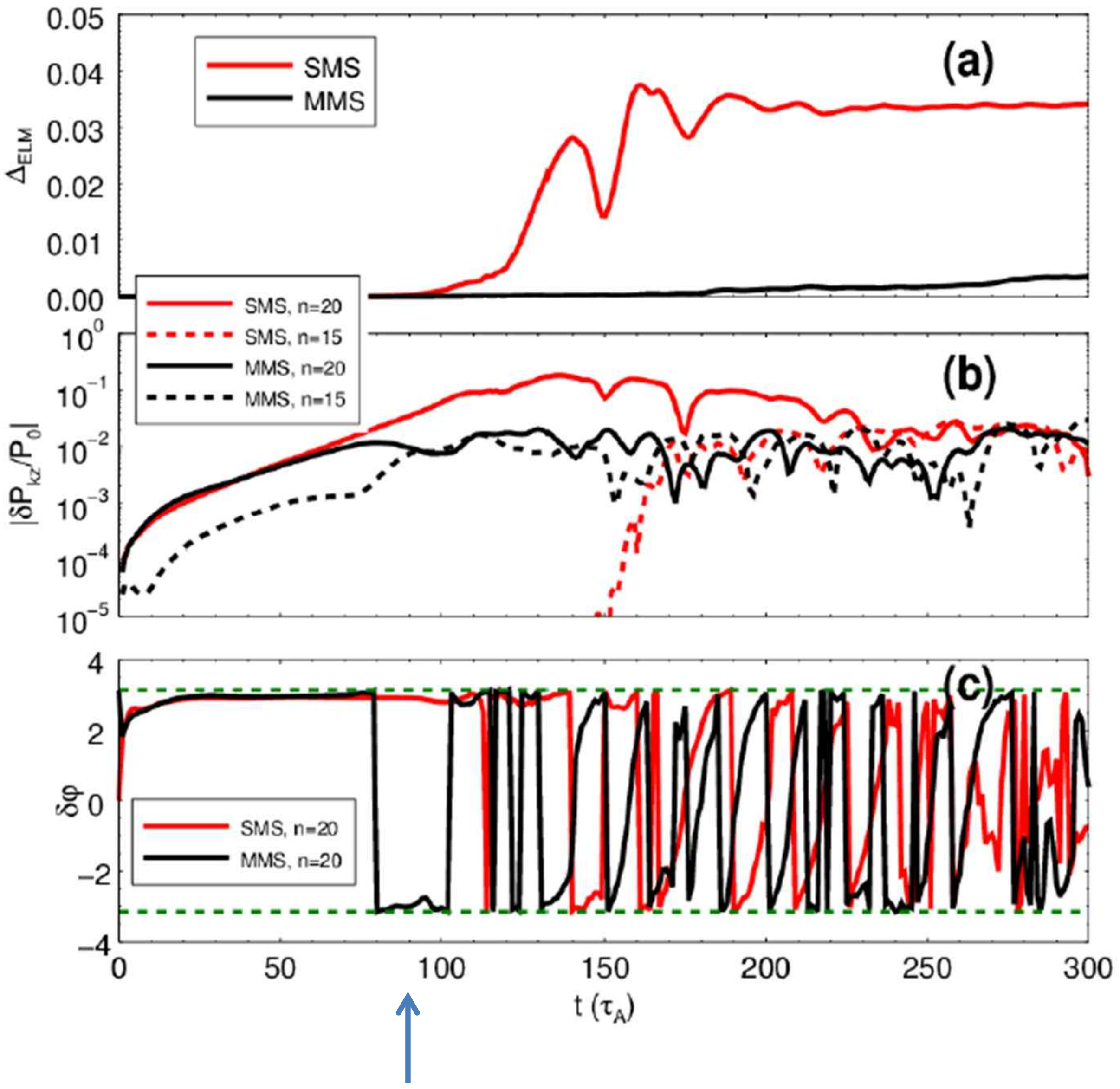
transfer → dissipation (weak)

Critical element: relative phase
$\delta\varphi = arg\ [\hat{p}_n\ /\ \hat{\phi}_n]$

Phase coherence time sets growth



Cross Phase Exhibits Rapid Variation in Multi-Mode Case



- Single mode case → coherent phase set by linear growth → rapid growth to ‘burst’
- Multi-mode case → phase de-correlated by mode-mode scattering → slow growth to turbulent state

Key Quantity: Phase Correlation Time

- Ala' resonance broadening (Dupree '66):

$$\frac{\partial}{\partial t}\hat{P} + \tilde{v}\cdot\nabla\tilde{P} + \langle v\rangle\cdot\nabla\hat{P} - D\nabla^2\hat{P} = -\tilde{v}_r\frac{d}{dr}\langle P\rangle$$

Nonlinear scattering

Linear streaming (i.e. shear flow)

Ambient diffusion

$$\hat{P} = Ae^{i\,\phi}$$

Relative phase ↔ cross-phase

Amplitude

$$\hat{v} = B$$

Velocity amplitude

N.B.: In essence, amplitude 'slaved' to phase

➔ $$\partial_t\tilde{\phi} + \tilde{v}\cdot\nabla\tilde{\phi} + \langle v(r)\rangle\cdot\nabla\tilde{\phi} - D\nabla^2\tilde{\phi} - \frac{2D}{A}\nabla A\cdot\nabla\tilde{\phi} = 0$$

NL scattering

shearing

$$\partial_t A + \tilde{v}\cdot\nabla A + \langle v(r)\rangle\cdot\nabla A + D\left(\nabla\tilde{\phi}\right)^2 A - D\nabla^2 A = -B\frac{d}{dr}\langle P\rangle$$

Damping by phase fluctuations

Phase Correlation Time

- Stochastic advection:

$$\frac{1}{\tau_{ck}} = \vec{k} \cdot D_\phi \cdot \vec{k} + k^2 D$$

$$D_\phi = \sum_{k'} \tau_{ck'} |\tilde{v}'_{\perp k}|^2$$

- Stochastic advection + sheared flow:

$$\frac{1}{\tau_{ck}} \approx \left(k_\perp^2 (D_\phi + D) \langle v_\perp \rangle'^{\,2}\right)^{1/3}$$

➔ Coupling of radial scattering and Shearing shortens phase correlation

➔ Strong $\langle V_E \rangle'$ beneficial

- Parallel conduction + diffusion:

$$\frac{1}{\tau_{ck}} \approx \left[\frac{\hat{s}^2 k_\perp^2}{(Rq)^2}\, \chi_\parallel \,(D_\phi + D)\right]^{1/2}$$

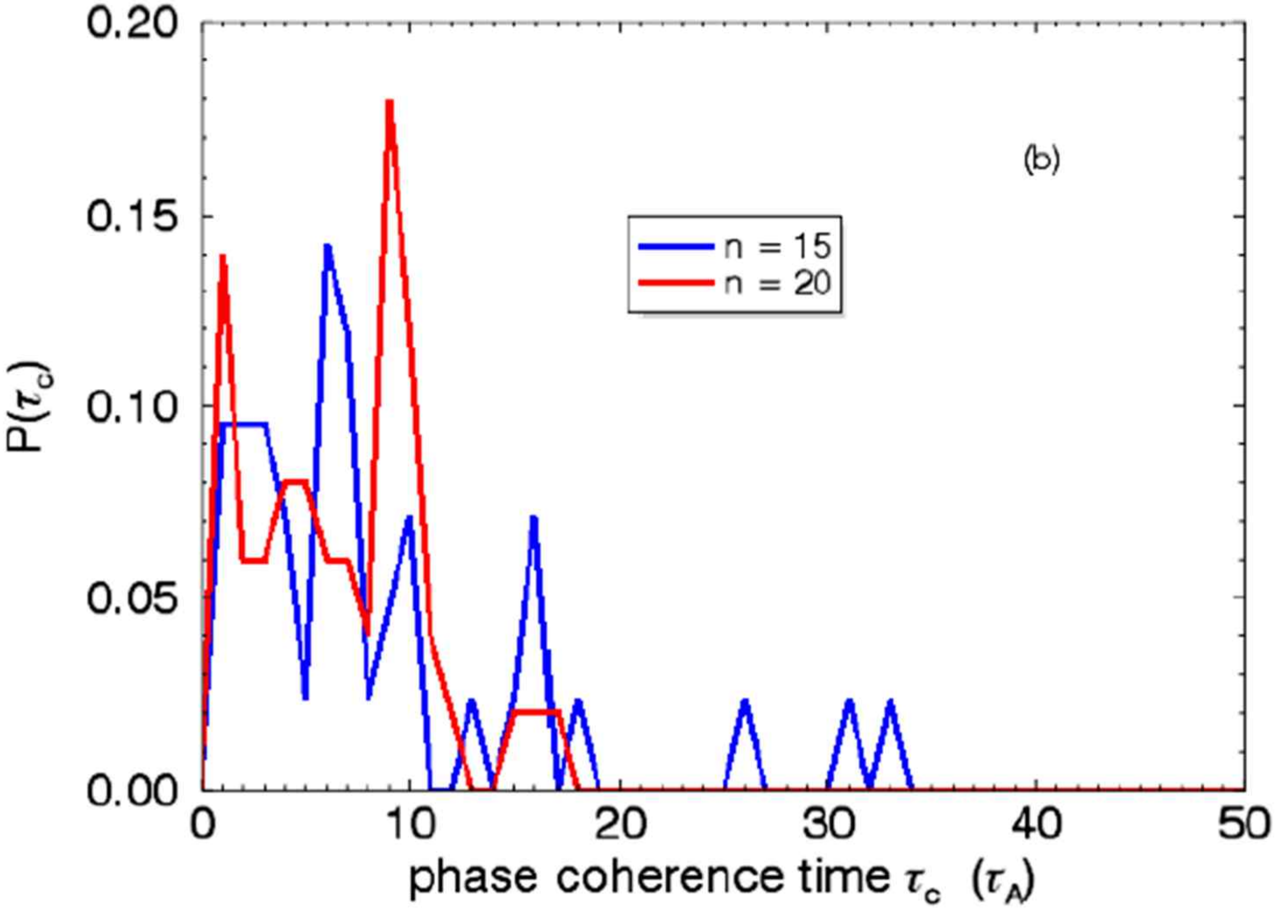
➔ Coupling of radial diffusion and conduction shortens phase correlation

What is actually known about fluctuations in relative phase?

- For case of P.-B. turbulence, a broad PDF of phase correlation times is observed. Further studies needed, especially 1) V_E effects, 2) EHO synergy

pdf

of τ_c



Implications for: i) Bursts vs Turbulence

ii) Threshold

Key: Peaked (coherent) vs Flat (stochastic)

growth spectrum

Bursts, Thresholds

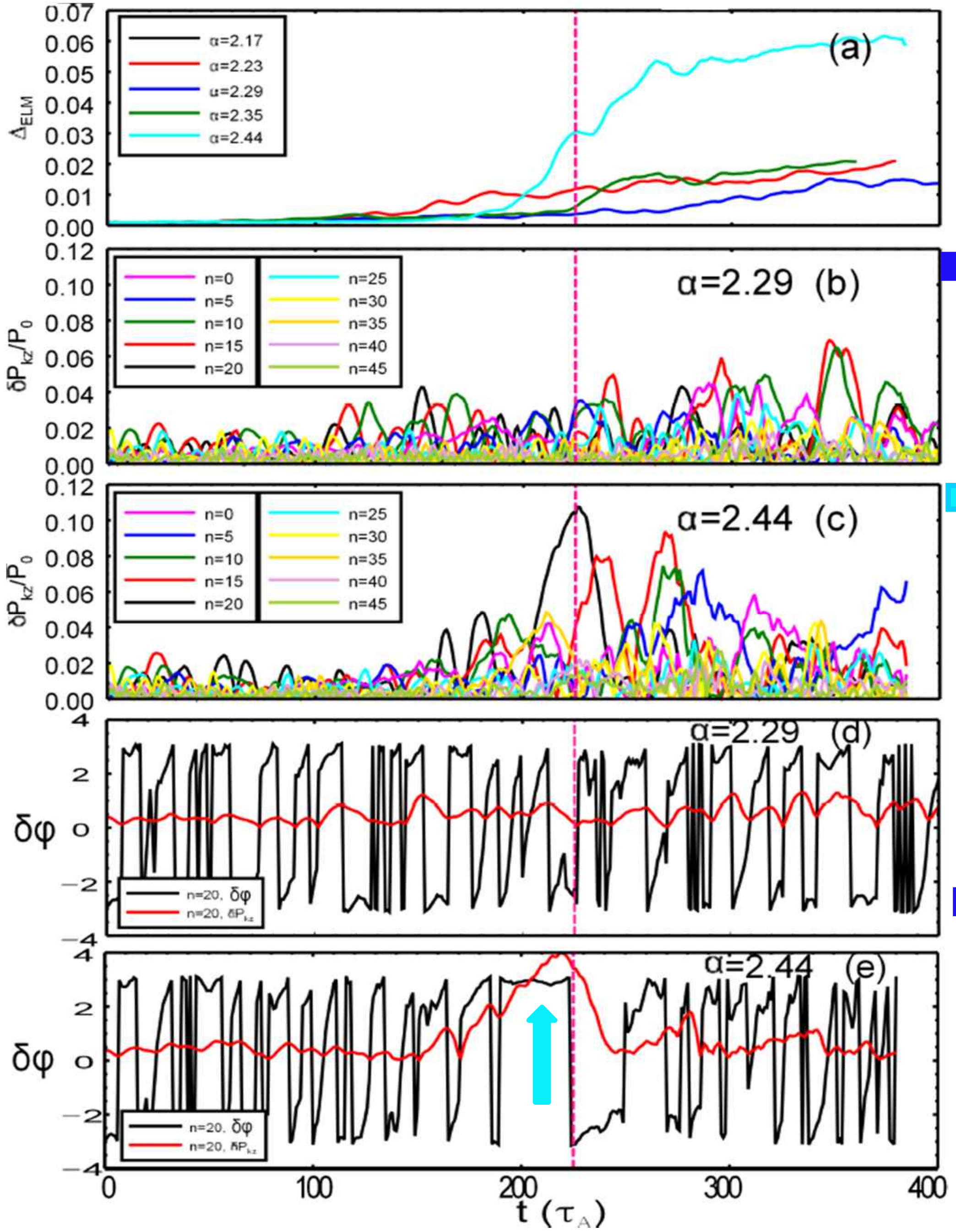
- P.-B. turbulence can scatter relative phase and so reduce/limit growth of P.-B. mode to large amplitude
- Relevant comparison is:

$$\gamma_k^L \text{ (linear growth) } \quad \text{vs} \quad \frac{1}{\tau_{ck}} \text{ (phase de-correlation rate)}$$

- Key point: Phase scattering for mode $\vec{k}$ set by 'background modes $\vec{k}'$' i.e. other P.-B.'s (or micro-turbulence) → where from?

➔ is the background strong enough?? ←→ profile of excitation!

The shape of growth rate spectrum determines burst or turbulence

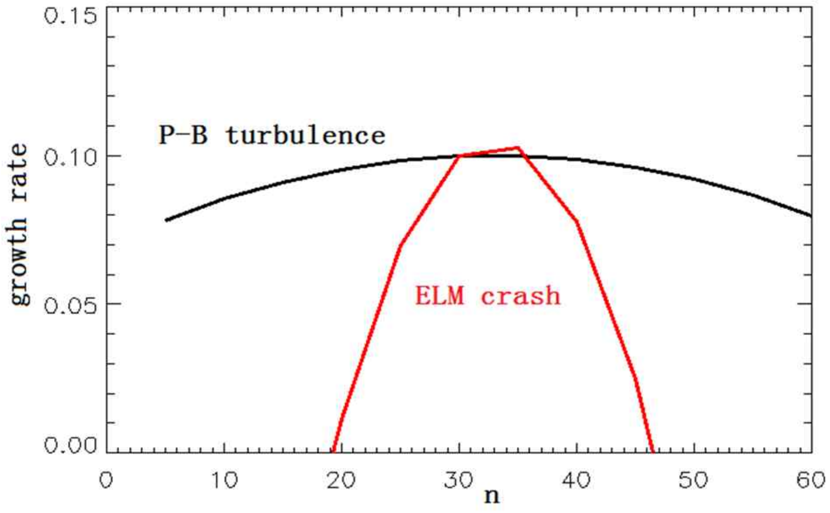


P-B turbulence

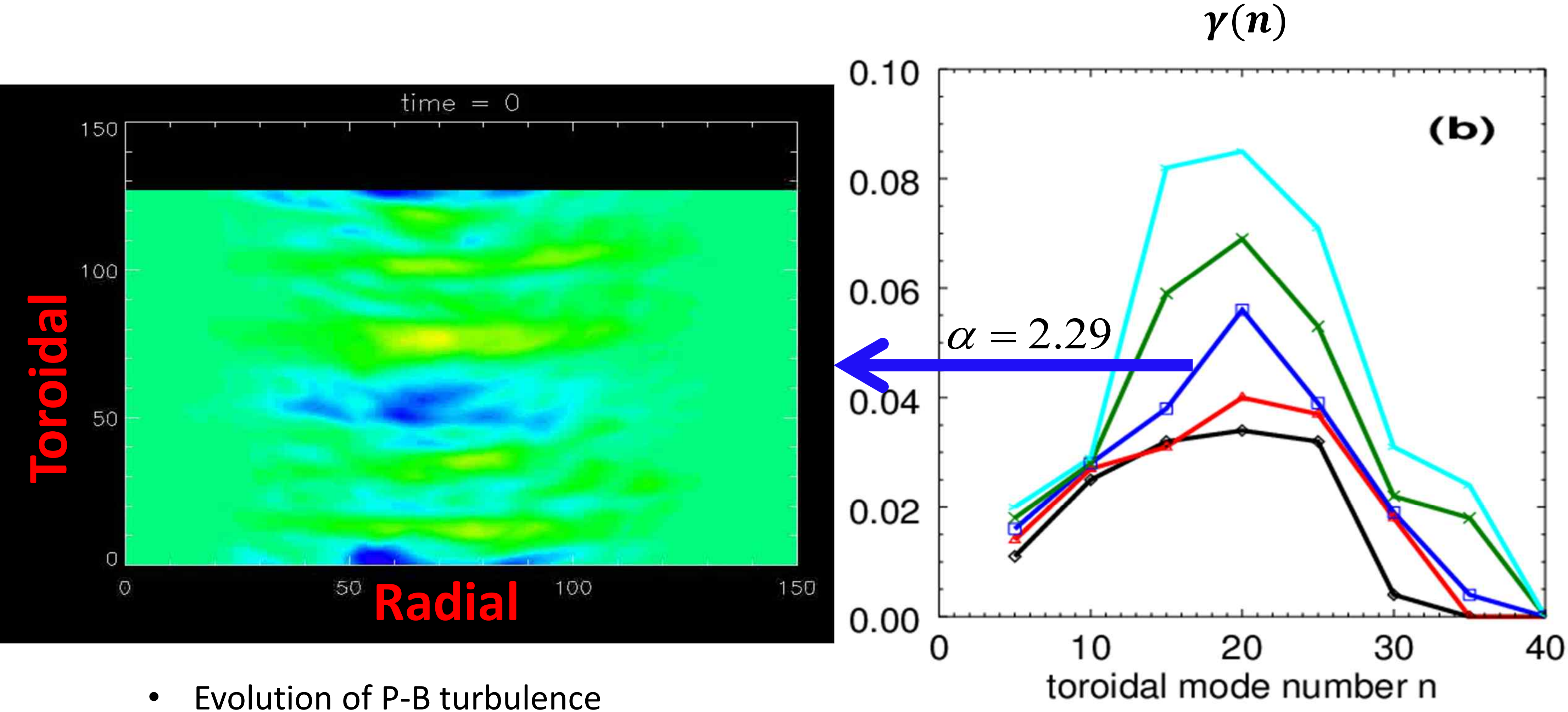
$$\gamma(n)\tau_c(n) < \ln 10$$

Isolated ELM crash

$$\begin{cases}\gamma(n)\tau_c(n) > \ln 10, n = n_{dom} \\ \gamma(n)\tau_c(n) < \ln 10, n \neq n_{dom}\end{cases}$$



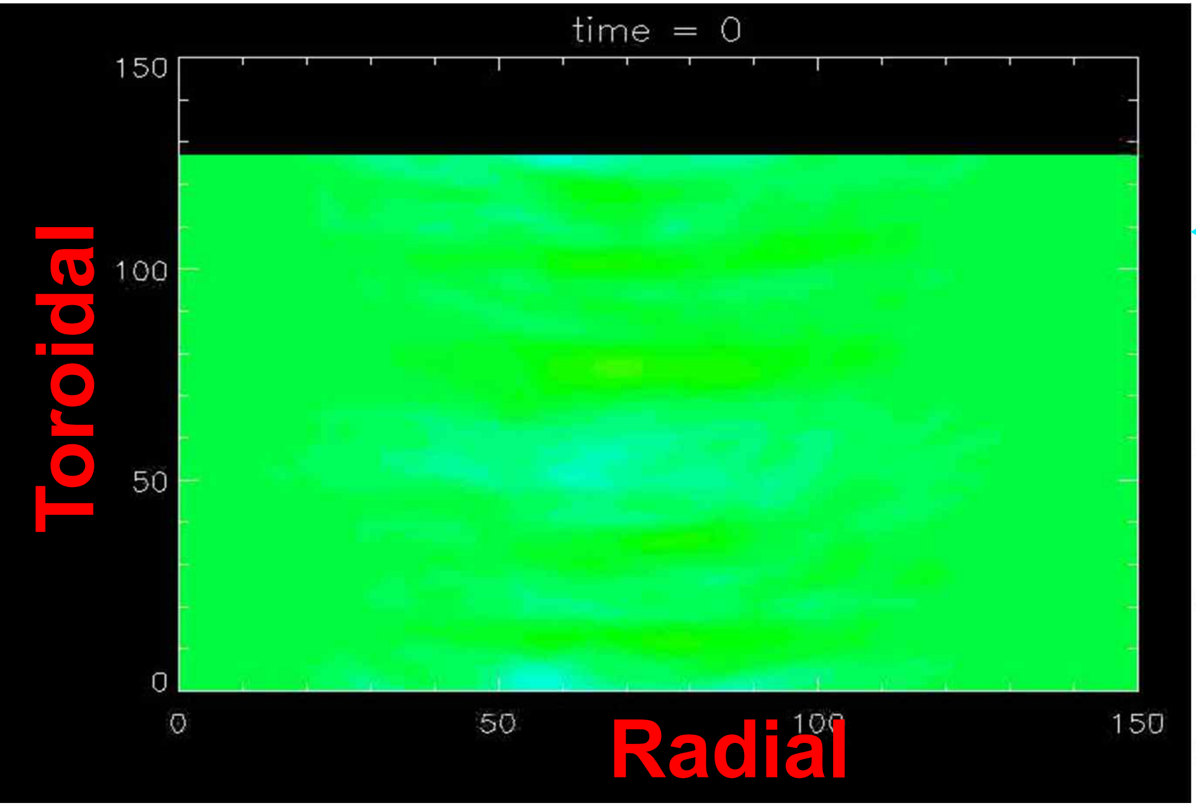
Modest $\gamma(n)$ Peaking ➔ P.-B. turbulence

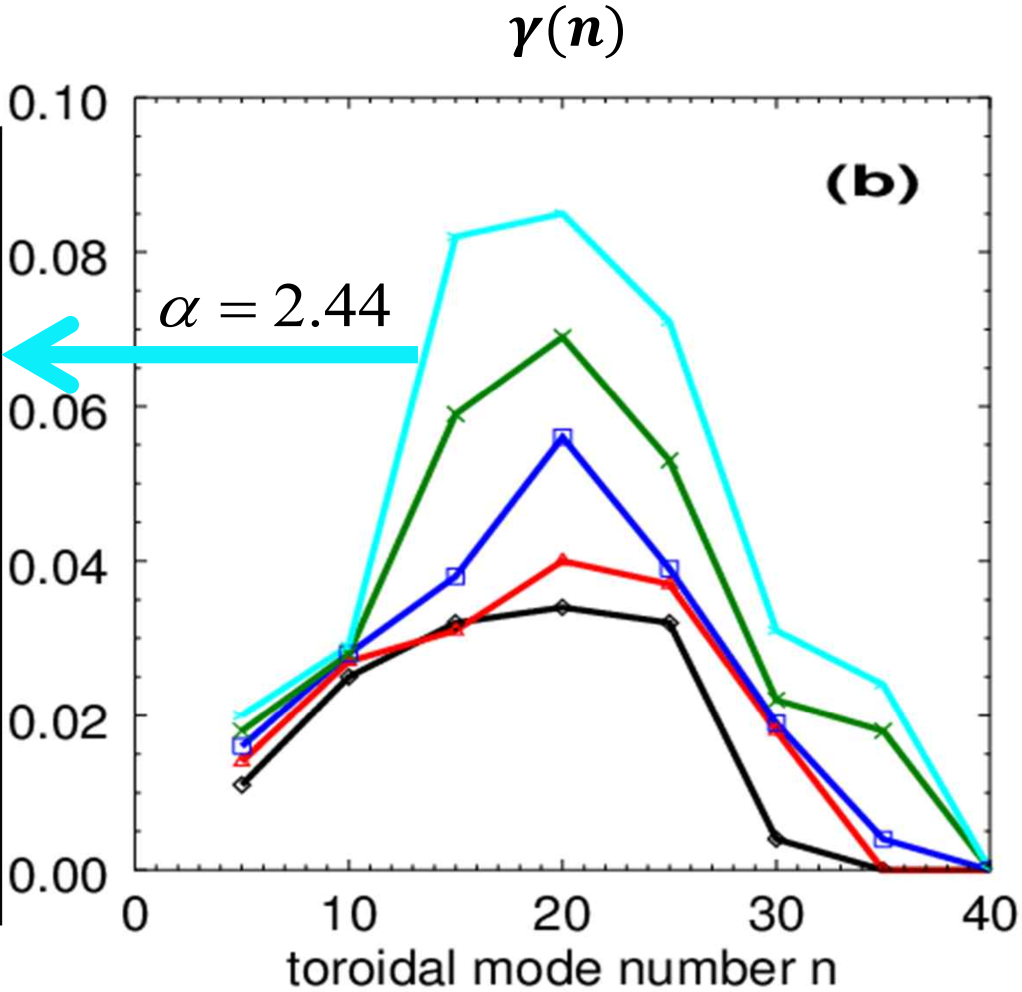


$$\boxed{\alpha = -2\mu_0 R P_0^{'} q^2 / B^2}$$

- Evolution of P-B turbulence
  - No filaments
  - Weak radial extent

Stronger Peaking $\gamma(n)$ ➔ ELM Crash

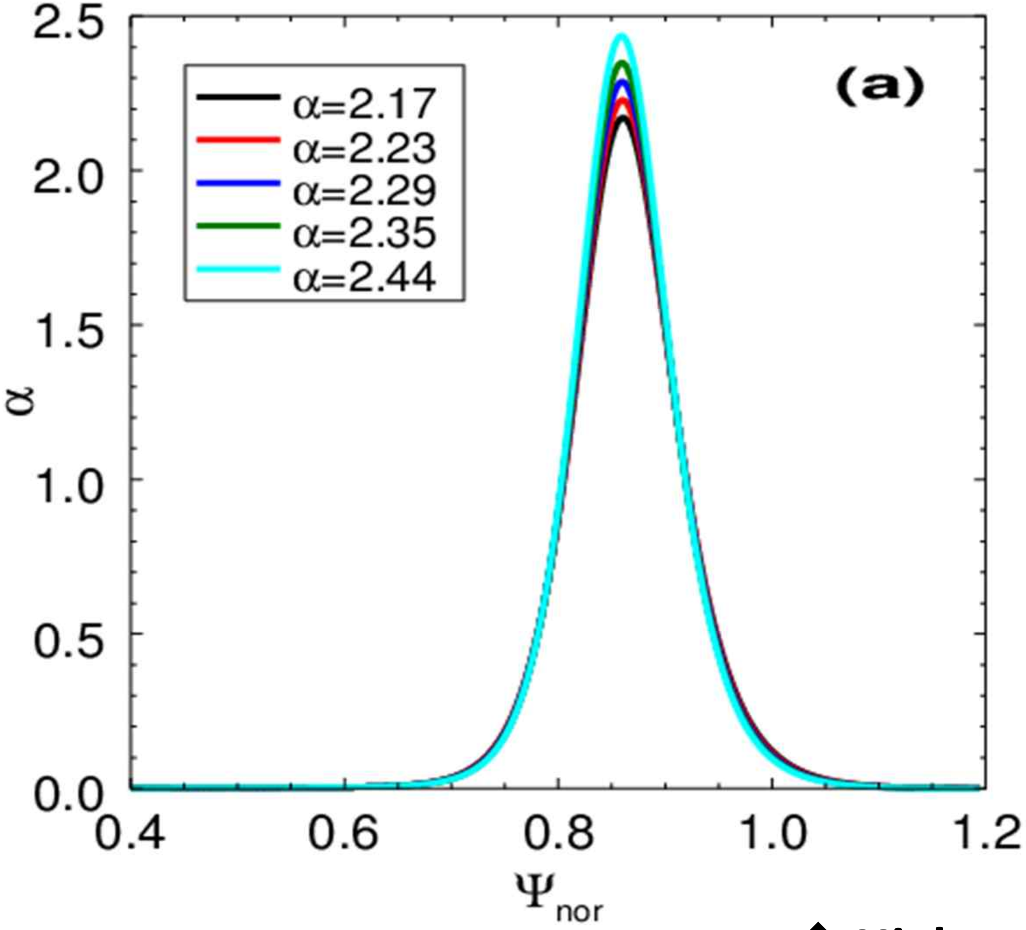




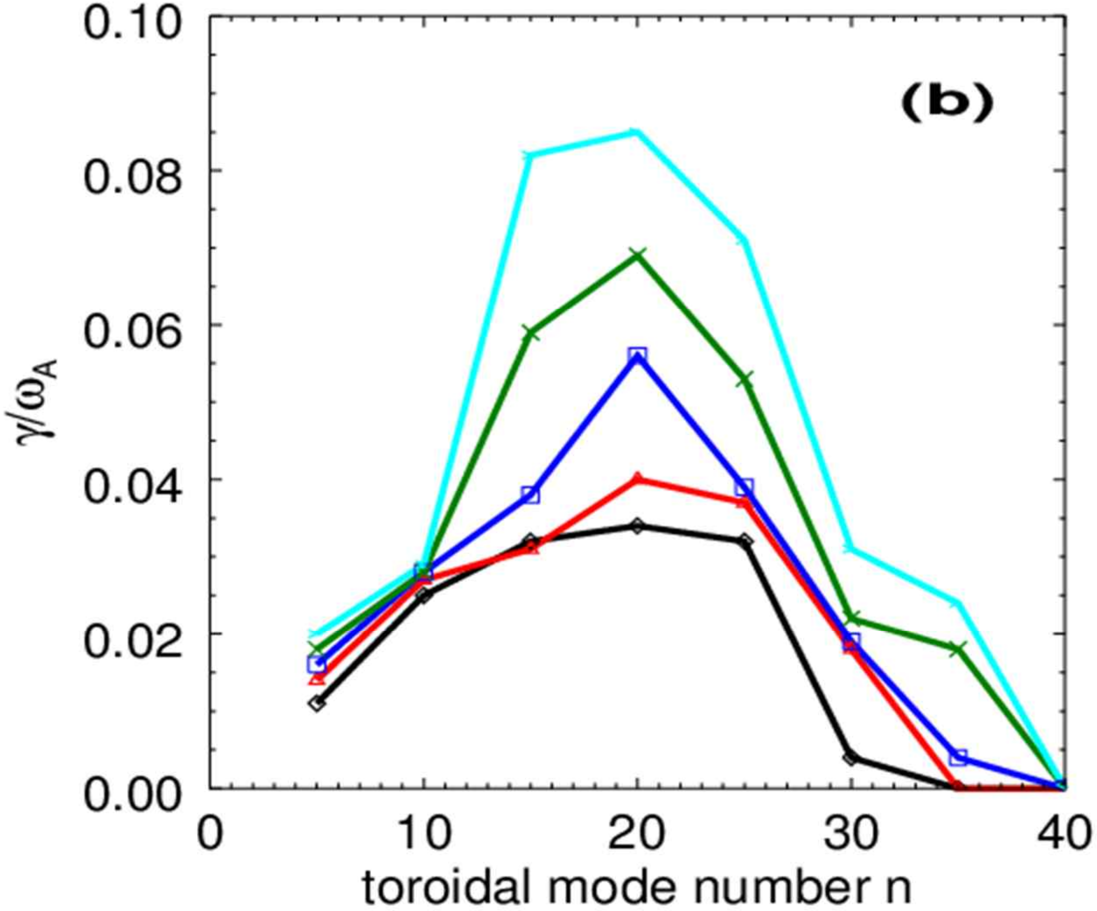
- ELM crash is triggered
- Wide radial extension

$$\boxed{\alpha = -2\mu_0 R P_0' q^2 / B^2}$$

$\gamma(n)$ Peaking VERY Sensitive to Pressure Gradient



Normalized pressure gradient



$\gamma(n)$

$$\boxed{\alpha = -2\mu_0 R P_0' q^2 / B^2}$$

◆ **Higher pressure gradient**

- ✓ Larger growth rate;
- ✓ **Peaking of growth rate spectrum**
  - → **disparity between peak, background?!**

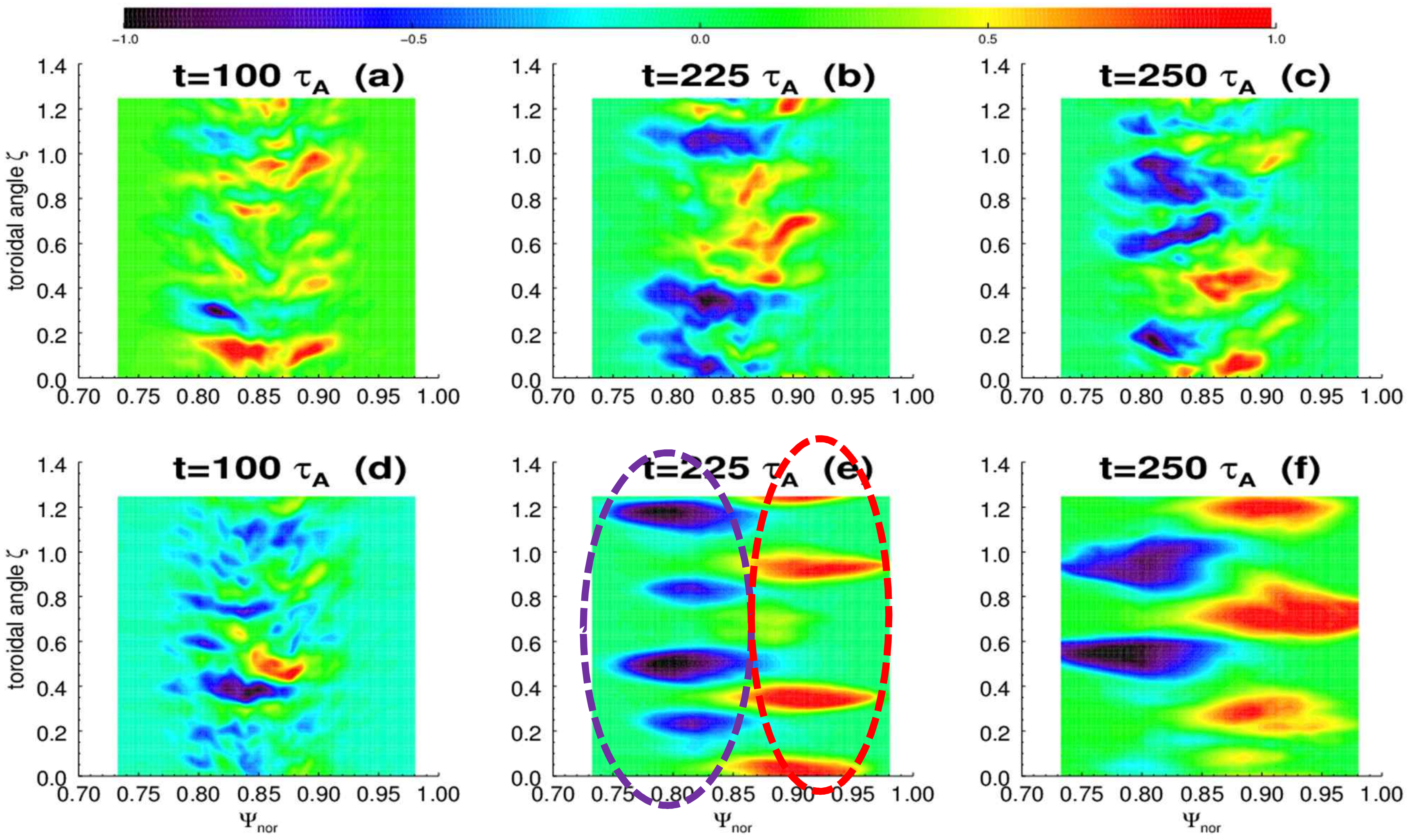
Filamentary structure may not correspond to that of the most unstable mode, due nonlinear interaction

$$\alpha = 2.29$$

P-B turbulence

$$\alpha = 2.44$$

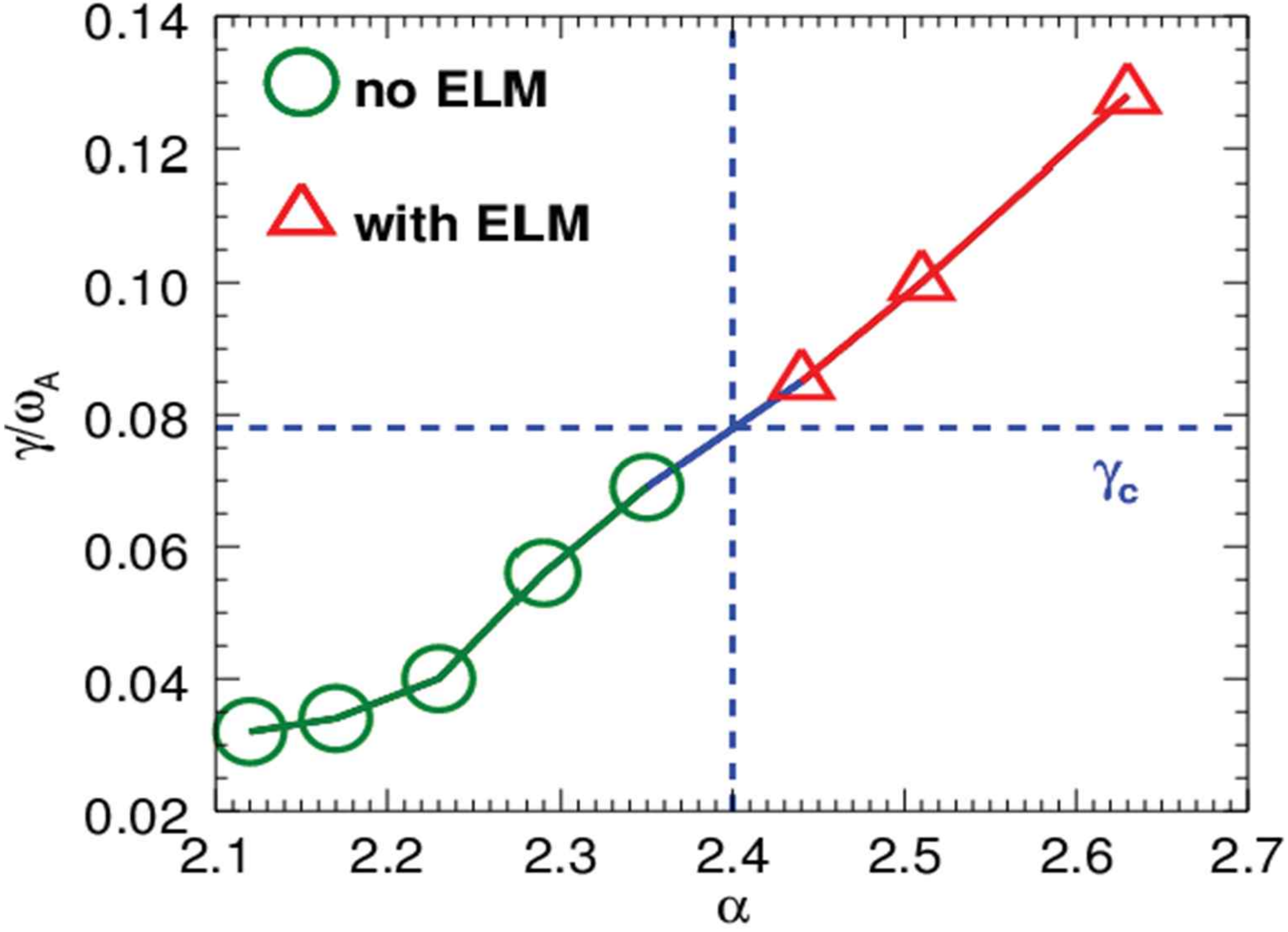
ELM crash



- ❑ **Triggering and the generation of filamentary structure are different processes!**
  - ✓ **ELM is triggered by the most unstable mode;**
  - ✓ **Filamentary structure depends on both linear instability and nonlinear mode interaction.**

Criterion for the onset of ELMs $\gamma > 0$ is replaced by the nonlinear criterion

$$\gamma > \gamma_c \sim 1/\,\tau_c$$



- **Criterion for the onset of ELMs**

$$\gamma\tau_c > \ln 10 \Rightarrow \gamma > \frac{\ln 10}{\tau_c} \equiv \gamma_c$$

- **Linear limit**

$$\lim_{\tau_c \to \infty} \Rightarrow \gamma > 0$$

- **γ_c is the critical growth rate which is determined by nonlinear phase scattering by background turbulence**
- **N.B. $1\ /\ \tau_c$ - and thus γ_{crit} - are functionals of $\gamma_L(n)$ peakedness**

Partial Summary

- Multi-mode P.-B. turbulence or ~ coherent filament formation can occur in pedestal
- Phase coherence time is key factor in determining final state and net P.-B. growth
- Phase coherence set by interplay of nonlinear scattering with ‘differential streaming’ in $\hat{P}$ response → V_E' highly favorable
- Key competition is γ_L vs $1 \,/\, \tau_c$ → defines effective threshold
- Peakedness of $\gamma(n)$ determines burst vs turbulence

- So, is this relevant to turbulent QH state?
- Appears yes:

Reconciles {
Turbulence (microscopic)
Absence of ELM burst/collapse

- Exploration of strong $\langle V_E \rangle'$ regimes should only strengthen case, by shortening τ_c by increased phase scattering
- Larger number of modes in turbulence increases phase scattering

Towards a Unified Scenario (?!)

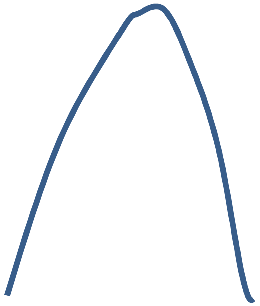
- Is there a connection between EHO/coherent and turbulent state?
- Elements:
  - $\langle V_E \rangle'$
  - Non-trivial phase evolution via $\left\{ \begin{array}{l} \text{Locking/slip} \\ \text{Scattering} \end{array} \right.$

i.e. $\frac{d\phi}{dt} = \overbrace{\underset{\text{shearing}}{k_\perp \Delta x}\, \langle V_E \rangle' - \underset{\text{drive}}{\frac{|\delta V|}{|\delta P|}} \langle p' \rangle \sin\phi} + \underset{\text{scattering}}{\delta\phi}$

- Really: locking vs scattering

- Considerations suggest:

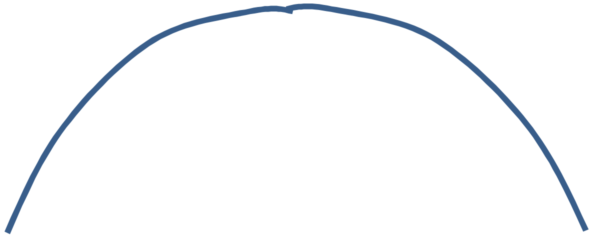
Peaked
$\gamma(n)$



➔ Regulate ϕ by $\omega_{E\times B}$ locking to MHD drive

→ $\left\{\begin{array}{l}\text{Phase slips, etc}\\ \text{coherent state}\end{array}\right.$

Broad
$\gamma(n)$



➔ Regulate ϕ by mode-induced scattering of ϕ, enhanced by V_E'

→ $\left\{\text{turbulent state}\right.$

- States are not exclusionary. May be synergistic.

So

- ExB shear can help by phase locking or/and phase scattering
- (Strongly) coherent and stochastic phase states are clear limits

←→ What macro control parameters set $\gamma(n)$ spectral structure?

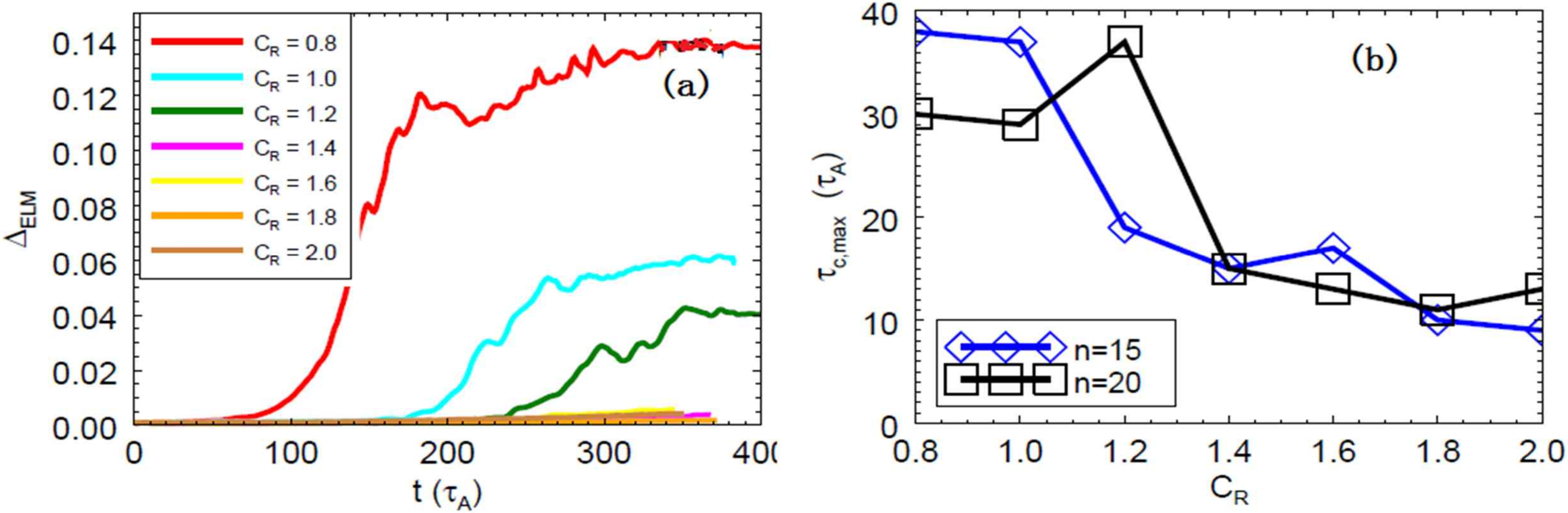
Things to Pursue

- Separate ExB effects i.e. Modelling + Experiment
  - Phase
  - Response Function
- Characterize turbulence/fluctuations in turbulent, LCO states
  - Long, $k_\perp \rho \ll 1$
  - Short, $k_\perp \rho \sim 1$ content
- Characterize phase scattering rates by fluctuation measurements i.e. $\delta\phi$ → τ_c, Δ etc. Is $k_\perp \rho < 1$ scattering sufficient to regulate PB? Or Is process multi-scale?
- Characterize transitions between different types OH → changes in fluctuations

BACK UP

ELMs can be controlled by reducing phase coherence time

$$\frac{\partial \varpi}{\partial t} + C_R \frac{\mathbf{b} \times \nabla \phi}{B} \cdot \nabla \varpi = RHS$$ i.e. scan C_R for fixed profiles



- ELMs are determined by the product $\gamma(n)\tau_c(n)$;
- Reducing the phase coherence time can limit the growth of instability;

- **Different turbulence states lead to different phase coherence times and, thus different ELM outcomes**

Keys to τ_c

- Scattering field
- ‘differential rotation’ in $\hat{P}$ response to $\hat{v}_r$

→ enhanced phase de-correlation

Knobs:	Mitigation States:
- ExB shear	- QH mode, EHO
- Shaping	- RMP
- Ambient diffusion	- SMBI
- Collisionality	- …